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STRUCTURE OF A SUPERSONIC JET WITH VARIED GEOMETRY OF THE NOZZLE ENTRANCE

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The structure of a supersonic underexpanded low-pressure-ratio jet exhausted from a nozzle with variable geometry of the entrance section is experimentally studied. Total pressure distributions in the initial cross sections of the examined jets are obtained. Based on these distributions, the coordinates of the mixing-region boundaries are found. The curvature of streamlines in the mixing layer within the first two barrels of an underexpanded jet is determined. A dependence generalizing the measurement results on the curvature of streamlines in the first barrel of a weakly underexpanded jet is obtained in dimensionless coordinates.

A supersonic underexpanded jet exhausted from an axisymmetric nozzle into an ambient space is characterized by significant nonuniformity in distributions of gas-dynamic quantities both in the axial and crossflow directions [1, 2]. Because of the nozzle-pressure ratio higher than unity, beginning from the nozzle-exit cross section, the velocity vector of the flow in a supersonic jet has a radial component directed away from the jet centerline. Consequently, the jet boundary at the initial section acquires a barrel-like shape. Further downstream, the radial component of the velocity vector changes its direction periodically, which favors the formation of a multicellular (multibarrel) structure of a weakly underexpanded jet [3, 4]. The streamlines close to the jet boundary acquire a certain curvature whose magnitude depends on the initial exhaust conditions: flow velocity, pressure difference at the nozzle exit, and initial state of the boundary layer of the jet.

A boundary layer is formed on the nozzle walls. Its thickness varies depending on the flow regime (Reynolds number Re) and nozzle geometry and roughness [5]. The change in the boundary-layer thickness leads to velocity and pressure redistribution not only near the walls but also in the vicinity of the nozzle-exit, which may become the reason for deviation of gas-dynamic quantities from preset values [6]. In supersonic jets, this leads to changes in the geometric size of the wave structure and, hence, the curvature of the jet boundary and streamlines in the mixing layer. The interest in this flow region is related to the Görtler instability, which leads to the emergence of streamwise vortex structures such as the Taylor–Görtler vortices [7]. Thus, the curvature of streamlines is an important factor in the formation of the Görtler instability in supersonic nonisobaric jets.

The objective of experimental studies was to determine the influence of the initial state of the boundary layer at the nozzle exit on gas dynamics of exhaustion of a weakly underexpanded jet and, primarily, on the curvature of the streamline of the mixing layer. The initial state of the boundary layer was varied by using nozzles with differently shaped entrance sections.

1. The experimental studies were performed with converging axisymmetric nozzles of two types: short and long nozzles consisting of permanent and replaceable parts (Fig. 1). (The quantities marked by the subscripts 1 and 2 refer to the short and long nozzles, respectively.) The permanent part BC of the nozzles with an exit diameter $D = 20$ mm had the shape of two adjoining cones with half-angles of 10 and 3°. The slope of the generatrix in the subsonic replaceable part AB of the short nozzle was 25°. The ratio of the diameters of the entrance and exit sections of the short nozzle was $D_1/D = 4.4$ and the relative length of the nozzle was $L_1/D = 6.45$. The replaceable part of the long nozzle $A'B$ was a cylindrical tube with a relative inner diameter $D_2/D = 1.28$ and length $L_2/D = 21.2$.

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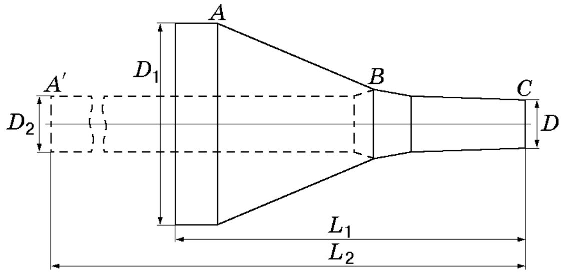

Fig. 1

The Reynolds number based on flow parameters at the exit and nozzle diameter was $\mathrm{Re}_d = 1.61 \cdot 10^6$. The object under study was a supersonic air jet with a stagnation temperature $T_0 = 283$ K. The exhausted jets were underexpanded, and the stagnation (total) pressure in the plenum chamber was $p_{\rm t} = 5 \cdot 10^2$ kPa. The quantities measured in the experiments were the Pitot pressure (by a standard cylindrical pressure gauge with an orifice diameter $d = 0.4$ mm and a fast-response inductive gauge) and the coordinate r of pressure-gauge displacement from the jet centerline. The distribution of the Pitot pressure along the jet radius was determined on the basis of measurement results obtained in various cross sections within the first three cells (barrels). The first cross section was located at a distance $x = 0.5R$ (R is the nozzle-exit radius). The values of $p_{\rm t}$ and r were measured by a data-acquisition system based on a personal computer connected to a CAMAC crate [8]. The error of pressure measurement was less than 1%.

2. Figure 2 shows the distributions of the relative Pitot pressure along the radius of jets exhausted from the short (dashed curves) and long (solid curves) nozzles. Figure 3 shows the profiles of the relative Pitot pressure at the jet centerline (in Figs. 3–5, the dashed curves and points 1 refer to the short nozzle, and the solid curves and points 2 refer to the long nozzle). Despite the fact that the supersonic jets were formed with an identical pressure in the plenum chamber, the Pitot pressure distributions along the radius and jet centerline are different. The greatest difference in pressure profiles $p_0(r)$ is observed in the first barrel ($x/R = 0.5$–4.0); these profiles for the second and third barrels almost coincide (the difference is observed at the jet axis only). Equalization of the profiles of gas-dynamic quantities at distant sections of the jets is caused by amplification of dissipative processes as the flow passes through a system of compression and expansion waves and by increasing influence of turbulence in the expanding mixing layer.

It follows from Fig. 3 that the pressure ratio ahead of and behind the normal shock at the exit cross section of the short nozzle is equal to unity, which is obtained for a Mach number $\mathrm{M}_1 = 1$. In the exit cross section of the long nozzle, we have $p_0/p_{\rm t} = 0.98$; hence, the jet-exhaustion velocity is supersonic: $\mathrm{M}_2 = 1.3$. To explain this result, we analyzed the changes in the boundary layer of both nozzles. The Reynolds numbers based on the parameters in the entrance cross sections of the short and long nozzles are $\mathrm{Re}_1 = 1.14 \cdot 10^5$ and $\mathrm{Re}_2 = 3.93 \cdot 10^5$, respectively. These values are typical of turbulent flow in nozzles. The flow velocity in the short nozzle varies from subsonic to the local velocity of sound in the throat, which coincides with the exit cross section. To characterize the reverse transition in a turbulent boundary layer of an accelerated flow, Moretti and Kays [9] proposed an acceleration parameter $K = (\nu/u^2)\, du/dx$ (ν is the kinematic viscosity; u and du/dx are the velocity and velocity gradient of the turbulent flow in the nozzle). At the section AB (see Fig. 1), the parameter is $K = 1.45 \cdot 10^{-5}$, which is several times the value at which structural reconstruction of the boundary layer begins. It is known that relaminarization of the turbulent boundary layer in a flow with a favorable streamwise pressure gradient begins already at $K = 2 \cdot 10^{-6}$ [10, 11]. As was noted in [6], in most cases, the boundary layer in converging nozzles can be considered as laminar. The characteristics of the boundary layer itself become different, in particular, the displacement thickness and momentum thickness, as well as the transverse component of turbulent fluctuations decrease significantly [12–15]. Becoming laminar, the boundary layer in a turbulent flow remains in this state in a gradient flow, i.e., relaminarization of the turbulent boundary layer "is delayed" to values of K close to zero. The displacement thickness calculated using the data of [11] is $\delta^* \approx 0.064$ mm. The velocity profile in the mixing layer near the exit section of a converging nozzle (actually, in the boundary layer at the nozzle exit) for $\mathrm{M} = 1$ corresponds to the Blasius profile [5].

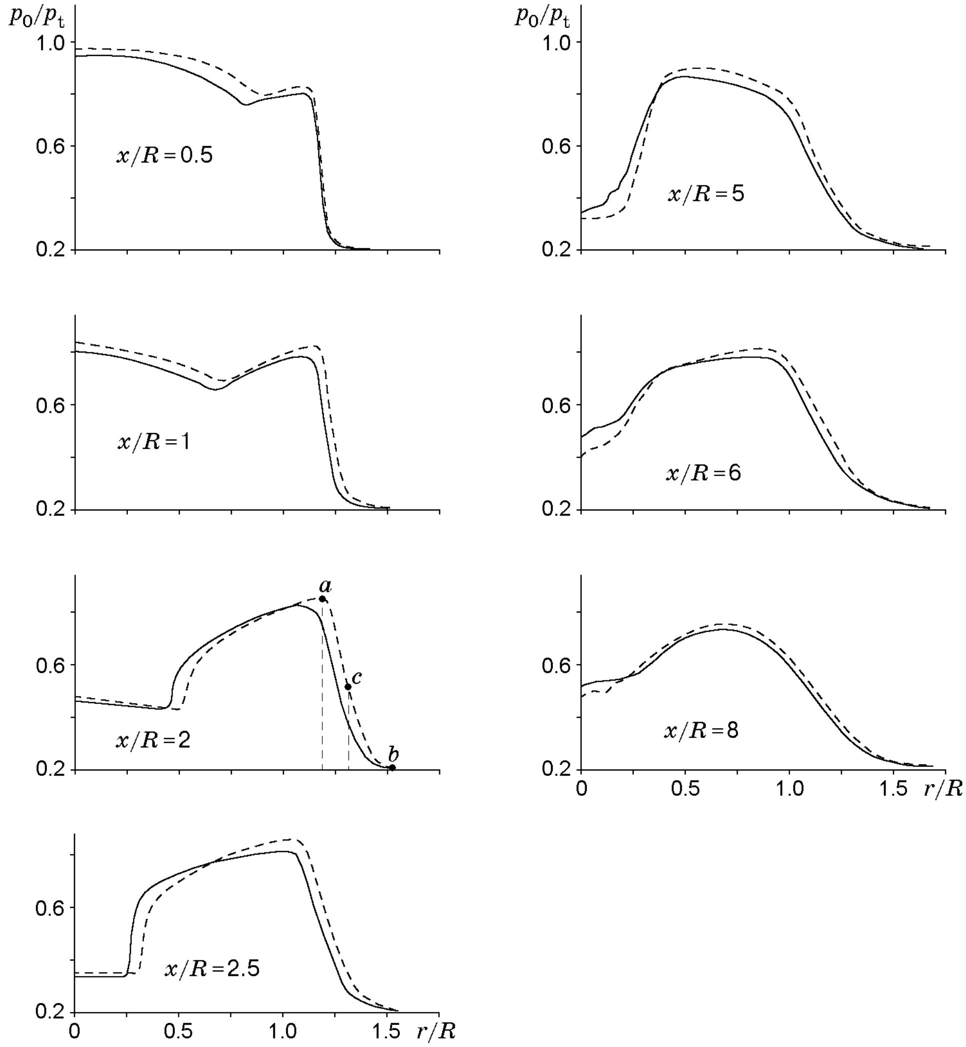
p_0/p_t
1.0
0.6
0.2
$x/R = 0.5$
$x/R = 1$
$x/R = 2$
a
c
b
$x/R = 2.5$
0
0.5
1.0
1.5
r/R
$x/R = 5$
$x/R = 6$
$x/R = 8$

Fig. 2

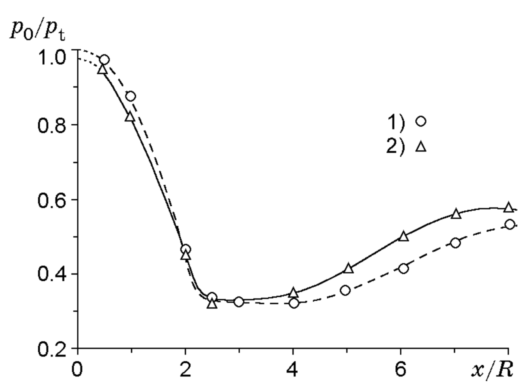
p_0/p_t
1.0
0.8
0.6
0.4
0.2
0
2
4
6
x/R
1) ○
2) △

Fig. 3

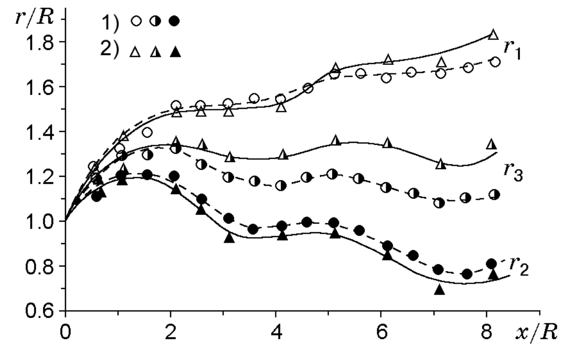

Fig. 4

The acceleration parameter at the converging part of the flow in the long nozzle is $K = 10^{-6}$. For this value of K, there are no structural changes in the turbulent boundary layer. At supercritical values of the pressure ratio, the boundary-layer displacement thickness reaches the maximum value $\delta^* = 0.148$ mm near the nozzle-exit section. Because of the pressure difference at the exit section of the long nozzle (difference in pressures in the jet and in the ambient space), the boundary-layer thickness decreases (the so-called boundary-layer escaping occurs); the reconstruction of the layer begins inside the nozzle, since the disturbances propagate upstream over the subsonic part of the flow. This leads to deformation of the boundary layer and displacement of the throat cross section inside the nozzle [6]. The so-called effective throat appears. The gas flow between the throat and exit cross sections is accelerated to a supersonic velocity.

Thus, the difference in gas-dynamic parameters at the exit of the short and long nozzles ($\mathrm{M}_1 = 1$ for $n = 2.65$ and $\mathrm{M}_2 = 1.3$ for $n = 1.84$) is responsible for the difference in results of pressure measurement in the jets.

Figure 4 shows the change in dimensionless streamwise and radial coordinates of the mixing layer of the initial section of the jets under study. The mixing layer is a region of a supersonic flow with a favorable crossflow pressure gradient, which is located between the fronts of the branched shock waves and jet boundary. The coordinate r_1 corresponds to the distance from the jet axis to a point with the maximum Pitot pressure in the profile $p_0(r)$ (point a in Fig. 2 for $x/R = 2$), the coordinate r_2 corresponds to the distance from the axis to the jet boundary (point b in Fig. 2), and the coordinate r_3 corresponds to the distance from the axis to a point where the Pitot pressure in the mixing layer is half the maximum value (point c in Fig. 2). The difference in the distances $(r_1 - r_2)/R$ is the relative thickness of the mixing layer of the jet. It follows from experimental data that the mixing layer thickness near the exit cross sections of the nozzles examined is approximately equal for different regimes of jet exhaustion. The layer thickness remains unchanged approximately up to the middle of the second barrel, and then (further downstream), the growth rate of the mixing layer thickness in a supersonic jet exhausted from the long nozzle increases. The curves $r_1(x)$ and $r_2(x)$ can be considered as the outer and inner boundaries of the layer, respectively. The wavy shape of the curves $r_1(x)$ and $r_2(x)$ is caused by periodic changes in the direction of the crossflow component of the velocity vector in a weakly underexpanded supersonic jet. The curve $r_3(x)$ is shifted toward the inner boundary for the short nozzle and is located almost symmetrically with respect to the boundary lines for the long nozzle, i.e., the Pitot pressure profiles $p_0(r)$ in the second case are more filled.

As was noted above, the curvature of streamlines plays the determining role in the development of the Görtler instability in supersonic jets. Within the first two or three barrels, the curvature of streamlines $r_3(x)$ changes its direction periodically and becomes either concave or convex with respect to the flow (Fig. 5). The curvature of streamlines in the mixing layer periodically changes its sign from negative (on the concave boundary) to positive (on the convex boundary). The curvature was calculated by the formula [16] $k = (d^2y/dx^2)/[1 + (dy/dx)^2]^{3/2}$, where $y = r_3/R$ is a dependence obtained by approximation of experimental data by a polynomial. Figure 5 shows the curvature of streamlines in the mixing layer of the examined jets as a function of the dimensionless coordinate $x/(R\mathrm{M}\sqrt{n})$. It is known that the streamwise size of supersonic axisymmetric jets is proportional to the parameter $\mathrm{M}\sqrt{\gamma n}$, where γ is the ratio of specific heats [17] (in the present experiment, $\gamma = 1.4$), and the crossflow size is proportional to the nozzle-pressure ratio of an underexpanded jet. The curvature of streamlines in the mixing layer depends on the nozzle-pressure ratio of a supersonic jet. The greater counterpressure experienced by the jet, the greater the curvature of streamlines. The maximum negative value of curvature is reached in the first

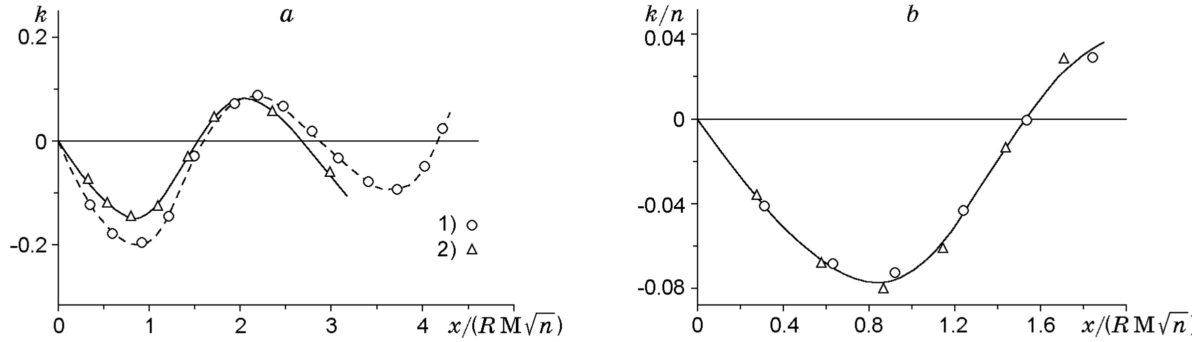

Fig. 5

barrel of the underexpanded jet, i.e., in the flow region where the Görtler instability is most pronounced. The first point of inflection of the curves ($k = 0$, the streamlines are parallel to the jet axis) in Fig. 5a corresponds to the maximum diameter of the jet boundary. Then, the sign of curvature changes to the opposite. Alternation of negative and positive values of curvature of streamlines in the mixing layer is repeated until the cellular structure of the nonisobaric jet degenerates due to dissipation.

Using the ratio of curvature to the nozzle-pressure ratio k/n, we can obtain a generalized dependence of the relative quantity r/n on the dimensionless streamwise coordinate $x/(R\text{M}\sqrt{n})$ within the first barrel of the jet (Fig. 5b).

The experimental studies show that the gas dynamics and structure of a weakly underexpanded supersonic jet exhausted from a converging nozzle depend on the initial state of the boundary layer in the exit cross section. The effect of geometry of the entrance section of the nozzle on the characteristics of the mixing region of supersonic jets was found.

Using the generalized dependence obtained, one can find the curvature of streamlines in the mixing layer of the first barrel of a weakly underexpanded supersonic jet in the first approximation for given gas-dynamic parameters of the flow and nozzle size.

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